Name Solutions

October 24, 2012

ECE 311 Exam 2 Fall 2012

Closed Text and Notes

1) Be sure you have 10 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(10 pts) 1. A conductor has cross-section of 0.01 m^2 and length 1 m. The density of the conduction electrons is $n=10^{20}\dfrac{electrons}{m^3}$. A dc voltage is applied and the electrons acquire an average drift velocity of $u=\dfrac{1}{1.6}\dfrac{m}{s}$. What current is flowing in the conductor?

$$J = \rho u = enu = \left(1.6\times10^{-19}\frac{C}{elec}\right)\left(10^{20}\frac{elec}{m^3}\right)\left(\frac{1}{1.6}\frac{m}{s}\right)$$

$$= 10\frac{A}{m^2}$$

$$I = JS = \left(10\frac{A}{m^2}\right)\left(0.01\,m^2\right)$$

$$I = 0.1\,A$$

(5 pts) 2. A 5V battery is connected to a parallel-plate capacitor. The battery is then disconnected without disturbing the charge on the capacitor plates. A dielectric with dielectric constant of 10 is placed between the plates. The potential difference between the plates is

A) 5 V
B) 0 V
C) 0.5 V
D) 50 V

$$D_{before} = D_{after}$$

$$E_{before} = \frac{D_{before}}{\epsilon_0}$$

$$E_{after} = \frac{D_{after}}{10\,\epsilon_0} = \frac{E_{before}}{10}$$

(5 pts) 3. A 5V battery is connected to a parallel-plate capacitor. The battery is then disconnected without disturbing the charge on the capacitor plates. The plates are now pulled apart to double the separation between the plates. The potential difference between the plates is

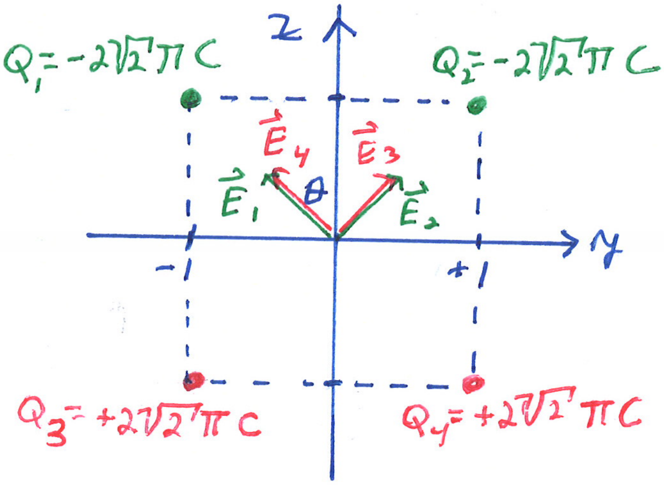
A) 5 V
B) 0 V
C) 2.5 V
D) 10 V

$$E_{after} = E_{before}$$

$$V_{after} = 2\,V_{before}$$

(10 pts) 4. Two point charges of $-2\sqrt{2}\pi C$ are located at (0, 1, 1) and (0, -1, 1). The xy plane is a grounded conducting plane.

(8 pts) A) what is $\rho_S(0,0,0)$? $D_n(0,0,0) = \rho_s(0,0,0)$ use the method of images to find $D_n(0,0,0)$



Shown are $\vec{E}_1(0,0,0)$, $\vec{E}_2(0,0,0)$, $\vec{E}_3(0,0,0)$ and $\vec{E}_4(0,0,0)$ caused by Q_1, Q_2, Q_3 and Q_4. There sum will only result in an $\hat{a}_z$ component with

$$\vec{E}(0,0,0) = 4E_1(0,0,0)\cos\theta\,\hat{a}_z$$

$$E_1(0,0,0) = \frac{2\sqrt{2}\pi C}{4\pi\epsilon_0(2m^2)} \qquad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\vec{E}(0,0,0) = 4\left[\frac{2\sqrt{2}\pi C}{4\pi\epsilon_0(2m^2)}\right]\frac{1}{\sqrt{2}}\hat{a}_z = \frac{1}{\epsilon_0}\hat{a}_z\frac{C}{m^2}$$

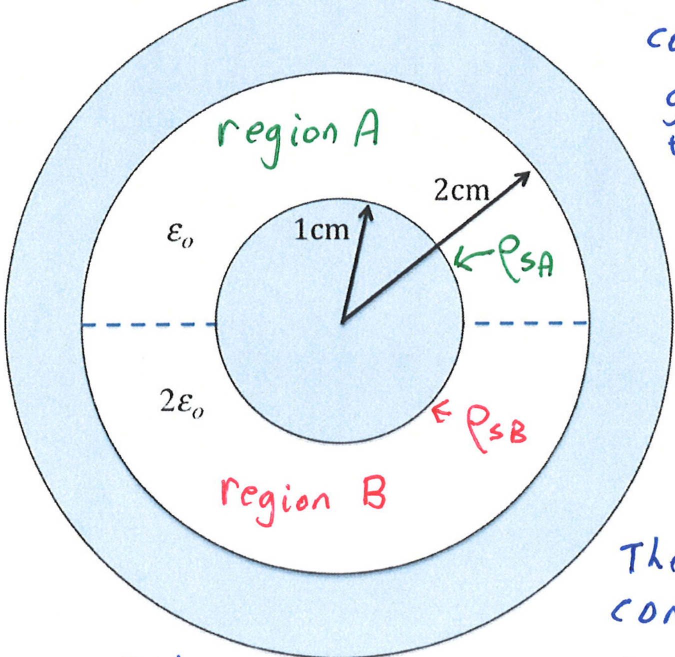
$$\vec{D}(0,0,0) = \epsilon_0\vec{E}(0,0,0) = 1\,\hat{a}_z\frac{C}{m^2}$$

$$\rho(0,0,0) = 1\frac{C}{m^2}$$

(2 pts) B) what is $\mathbf{E}(0, 0, -5m)$

0 since the location is below the grounded plane

(15 pts) 5. A capacitor consists of two co-centric cylinders, shown is the cross-section. As shown, half of the region between the cylinders is filled with a dielectric of permittivity ε_o and half with a dielectric of permittivity $2\varepsilon_o$. If the length of the structure is 1 m, determine the capacitance.



Place a voltage V across the two conductors with the outer conductor grounded. Find the resulting Q on the inner conductor to obtain $C = \frac{Q}{V}$

The electric field intensity has to be the same in regions A and B so that

$$V = -\int_{\rho=0.02m}^{\rho=0.01m} E\, d\rho$$

The form of $\vec{E}$ for a cylindrical conductor is $\vec{E} = \frac{k}{\rho}\hat{a}_\rho$

$$V = -\int_{0.02}^{0.01} \frac{k}{\rho}\, d\rho = -k \ln\rho \Big|_{0.02}^{0.01} = -k[\ln(0.01) - \ln(0.02)] = k \ln 2$$

so, $\vec{E} = \frac{V}{\rho \ln 2}\hat{a}_\rho$

since $\vec{D} = \epsilon \vec{E}$

$$\vec{D}_A = \epsilon_o \vec{E} = \frac{\epsilon_o V}{\rho \ln 2}\hat{a}_\rho \qquad \vec{D}_B = 2\epsilon_o \vec{E} = \frac{2\epsilon_o V}{\rho \ln 2}\hat{a}_\rho$$

$$\rho_{sA} = \frac{\epsilon_o V}{(0.01)\ln 2} = \frac{100\epsilon_o V}{\ln 2} \qquad \rho_{sB} = \frac{2\epsilon_o V}{(0.01)\ln 2} = \frac{200\epsilon_o V}{\ln 2}$$

area of inner conductor $= 2\pi\rho L = 2\pi(0.01m)(1m) = 0.02\pi\ m^2$

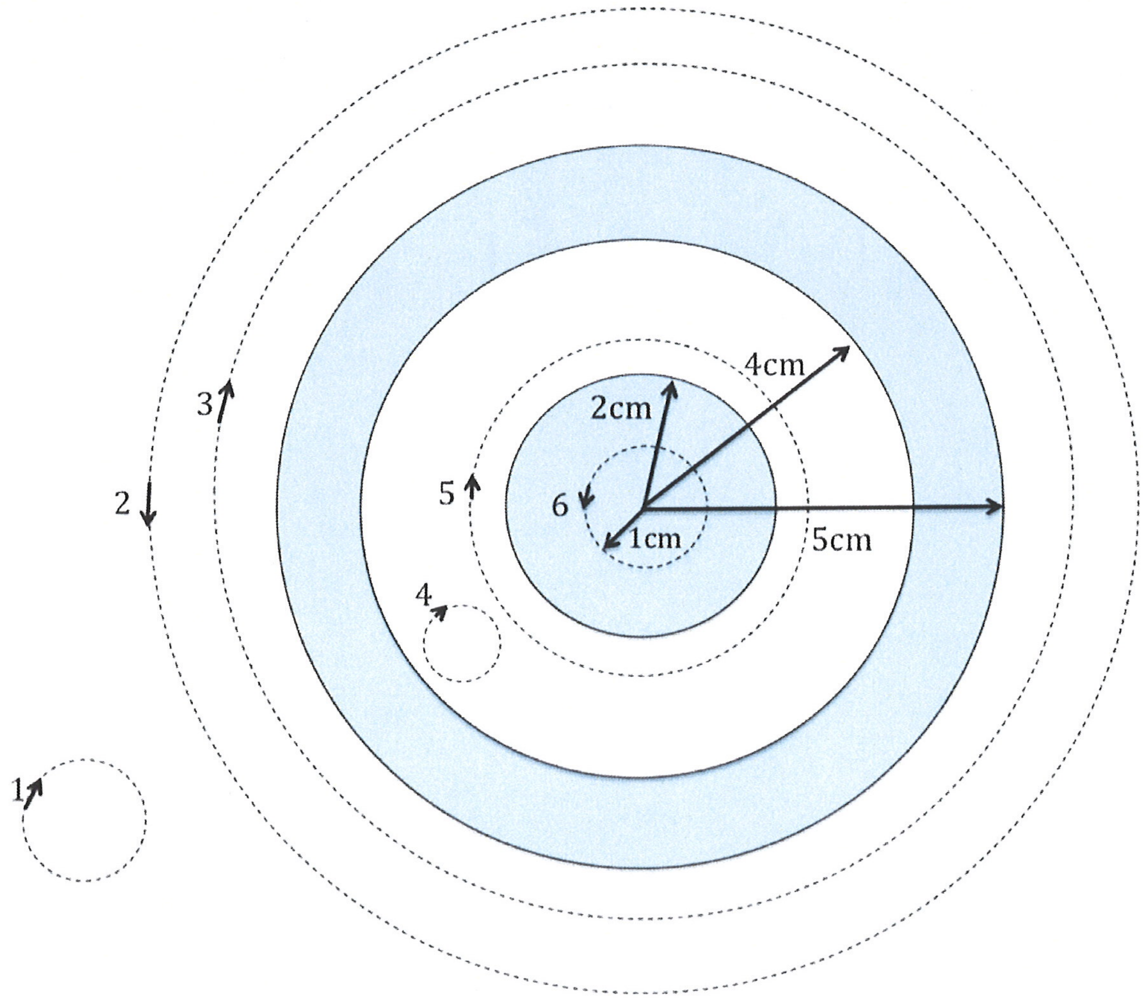
$$Q = \rho_{sA}(\tfrac{1}{2}\text{ area of inner cylinder}) + \rho_{sB}(\tfrac{1}{2}\text{ area of inner cylinder})$$

$$= \frac{100\epsilon_o V}{\ln 2}(0.01\pi) + \frac{200\epsilon_o V}{\ln 2}(0.01\pi) = \frac{3\epsilon_o V\pi}{\ln 2}$$

$$C = \frac{Q}{V} = \frac{3\epsilon_o \pi}{\ln 2}$$

(6 pts) 6. Shown are two conductors. The inner conductor is a solid cylinder of radius 2 cm. The outer conductor is a hollow cylinder of inner radius 4 cm and outer radius 5 cm. A current of 1 A is flowing into the page uniformly across the inner solid cylinder and 1 A out of the page uniformly across the outer hollow cylinder.

Determine $\oint \mathbf{H} \bullet \mathbf{dl}$ over the dashed paths shown.



$\oint_1 \mathbf{H} \bullet \mathbf{dl} =$ 0

$\oint_2 \mathbf{H} \bullet \mathbf{dl} =$ 0

$\oint_3 \mathbf{H} \bullet \mathbf{dl} =$ 0

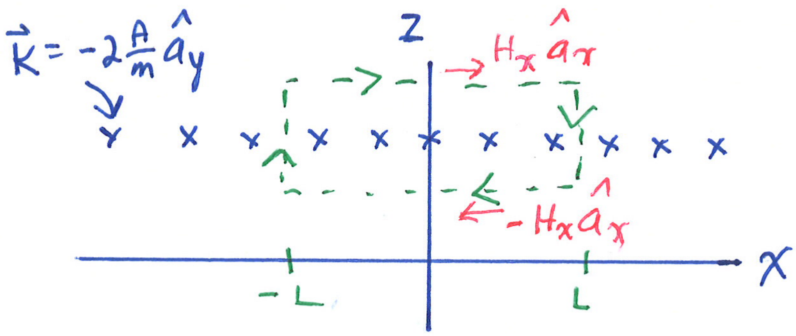
$\oint_4 \mathbf{H} \bullet \mathbf{dl} =$ 0

$\oint_5 \mathbf{H} \bullet \mathbf{dl} =$ 1 A

$\oint_6 \mathbf{H} \bullet \mathbf{dl} =$ − 0.25 A

(12 pts) 7. Two infinite conducting planes are situated at z =1m and z = -1m. On the z = 1m plane is flowing a sheet current density of $\mathbf{K}=2\frac{A}{m}\hat{\mathbf{a}}_y$, and on the z = -1 m plane is flowing a sheet current density of $\mathbf{K}=-2\frac{A}{m}\hat{\mathbf{a}}_y$. Find the magnetic field intensity everywhere.

Let's first find the magnetic field intensity caused by the $\vec{K} = -2\frac{A}{m}\hat{a}_y$ sheet current density at $z = 1m$



From the symmetry,

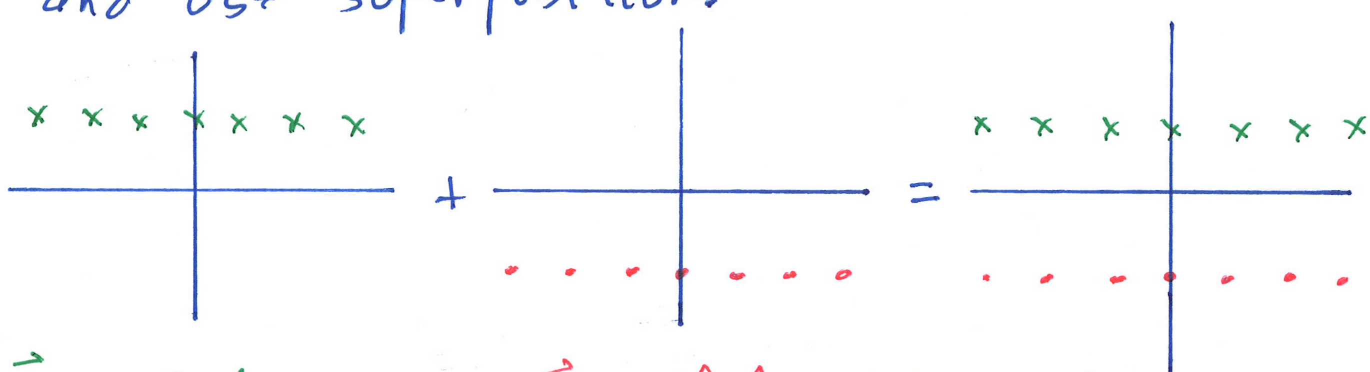
$$\vec{H} = H_x\hat{a}_x \quad z > 1m$$
$$-H_x\hat{a}_x \quad z < 1m$$

Apply Ampere's Circuit Law to the path shown

$$\oint \vec{H}\cdot d\vec{l} = H_x 2L + H_x 2L = I_{enclosed} = K2L = \left(2\frac{A}{m}\right)2L = 4H_xL$$

$$H_x = 1\frac{A}{m}$$

From this result we can infer the magnetic field intensity for the $z = -1m$ sheet current density and use superposition.



$$\vec{H} = 1\frac{A}{m}\hat{a}_x, \; z > 1m$$
$$-1\frac{A}{m}\hat{a}_x, \; z < 1m$$

$+$

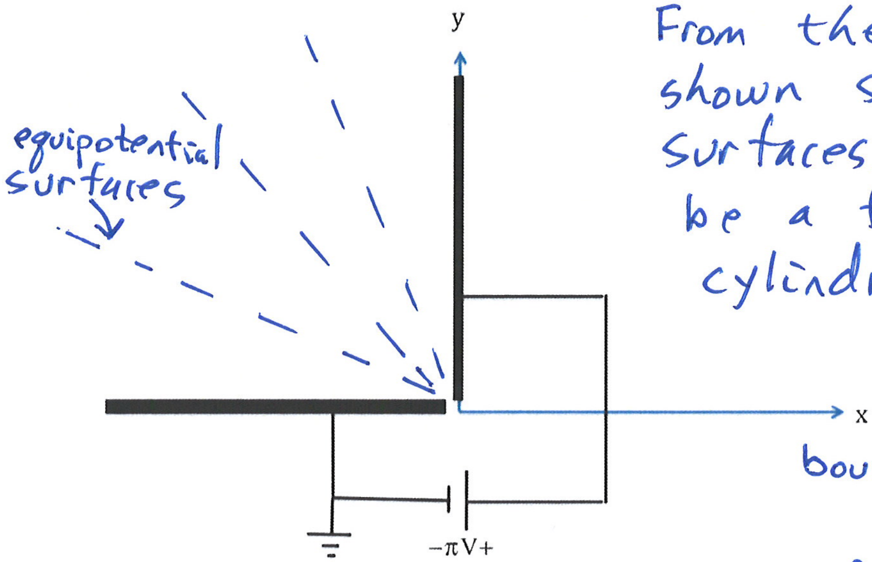
$$\vec{H} = -1\frac{A}{m}\hat{a}_x, \; z > -1m$$
$$1\frac{A}{m}\hat{a}_x, \; z < -1m$$

$=$

$$\vec{H} = 0 \quad z > 1m$$
$$-2\frac{A}{m}\hat{a}_x, \; -1m < z < 1m$$
$$0 \quad z < -1m$$

(14 pts) 8. Shown are two conductors. One is the semi-infinite half-plane at $\phi=\frac{\pi}{2}$ and the other is the semi-infinite half-plane at $\phi=\pi$. The region $\frac{\pi}{2}<\phi<\pi$ is free space.

(10 pts) A) Find the potential for $\frac{\pi}{2}<\phi<\pi$.



From the symmetry are shown some equipotential surfaces. So V will only be a function of ϕ with cylindrical symmetry.

$\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$, but $\rho_v = 0$

boundary conditions $V(\frac{\pi}{2}) = \pi V$
$V(\pi) = 0$

$$\nabla^2 V = 0 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\Big(\rho \underbrace{\frac{\partial V}{\partial \rho}}_{=0}\Big) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{=0}$$

$$\frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial V}{\partial \phi} = A \Rightarrow V(\phi) = A\phi + B$$

$$V(\pi) = 0 = A\pi + B \quad \Rightarrow B = -A\pi$$

$$V(\phi) = A\phi - A\pi$$

$$V(\tfrac{\pi}{2}) = \pi V = A\tfrac{\pi}{2} - A\pi = -A\tfrac{\pi}{2} \Rightarrow A = -2V, \; B = 2\pi V$$

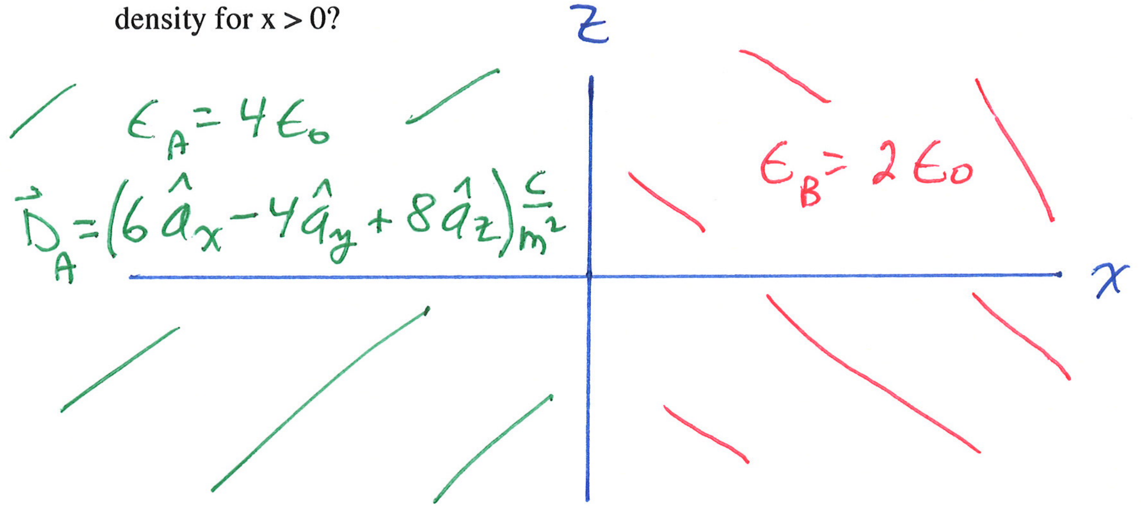
$$V(\phi) = (-2\phi + 2\pi)\,V$$

(4 pts) B) Find the electric field intensity for $\frac{\pi}{2}<\phi<\pi$.

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial \rho}\hat{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi - \frac{\partial V}{\partial z}\hat{a}_z$$

$$\vec{E} = -\frac{1}{\rho}\frac{\partial}{\partial \phi}\left[(-2\phi + 2\pi)V\right]\hat{a}_\phi = \frac{2V}{\rho}\hat{a}_\phi$$

(9 pts) 9. For the region x < 0 the electric flux density is $\mathbf{D}=(6\hat{\mathbf{a}}_x - 4\hat{\mathbf{a}}_y + 8\hat{\mathbf{a}}_z)\frac{C}{m^2}$ and $\varepsilon=4\varepsilon_o$. There is a charge density on the x = 0 plane of $\rho_S=2\frac{C}{m^2}$. If $\varepsilon=2\varepsilon_o$ for x > 0, what is the electric flux density for x > 0?



$\vec{D}_{nA} = 6\hat{a}_x \frac{C}{m^2}$ $\quad \vec{D}_{TB} = (-4\hat{a}_y + 8\hat{a}_z)\frac{C}{m^2}$

$\vec{E}_{TB} = \frac{D_{TB}}{4\epsilon_o} = \frac{1}{4\epsilon_o}(-4\hat{a}_y + 8\hat{a}_z)\frac{C}{m^2}$

Now apply boundary condition at $x=0$

$D_{nB} - D_{nA} = \rho_s \qquad D_{nB} - 6\frac{C}{m^2} = 2\frac{C}{m^2} \qquad D_{nB} = 8\frac{C}{m^2}$

$\vec{E}_{TA} = \vec{E}_{TB} = \frac{1}{4\epsilon_o}(-4\hat{a}_y + 8\hat{a}_z)\frac{C}{m^2}$

$\vec{D}_{TB} = 2\epsilon_o \vec{E}_{TB} = \frac{2\epsilon_o}{4\epsilon_o}(-4\hat{a}_y + 8\hat{a}_z)\frac{C}{m^2} = (-2\hat{a}_y + 4\hat{a}_z)\frac{C}{m^2}$

$\vec{D}_B = (8\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z)\frac{C}{m^2}$

(14 pts) 10. Fill in the table with the units, and whether a vector or scalar, for indicated quantities.

	units	Vector or scalar?
Electric Field Intensity	$\frac{V}{m}$	vector
Electric flux density	$\frac{C}{m^2}$	vector
Electric flux	C	scalar
potential	V or $\frac{J}{C}$	scalar
Magnetic field intensity	$\frac{A}{m}$	vector
Magnetic flux density	$\frac{Wb}{m^2}$ or T	vector
Magnetic flux	Wb or Vs	scalar